Calculate the compressibility factor and the molar volume of methanol steam at 200°C and 10 bar

a) using the ideal gas law
b) with the virial equation truncated after the 3rd virial coefficient

\[ v_m := \frac{R \cdot T}{P} \]
\[ v_m = 3.934 \text{ mol}^{-1} \text{ dm}^3 \]
\[ z = 1 \text{ by definition} \]

Solution by using the virial equation:

\[ \frac{P \cdot v}{R \cdot T} = 1 + B \cdot \frac{1}{v} + C \left( \frac{1}{v} \right)^2 \]

The tolerance has to be decreased for receiving a more precise result.

\[ \text{TOL} := 10^{-18} \]

For solving the equation the root-function is used. Finding the root requires a starting value for the volume:

\[ v := \frac{R \cdot T}{P} \quad v = 3.934 \text{ dm}^3 \text{ mol}^{-1} \]

\[ v_m := \text{root} \left[ \frac{P \cdot v}{R \cdot T} - 1 - B \cdot \frac{1}{v} - C \left( \frac{1}{v} \right)^2, v \right] \]
\[ v_m = 3.695834 \text{ dm}^3 \text{ mol}^{-1} \]
\[ z := \frac{P \cdot v_m}{R \cdot T} \quad \Rightarrow \quad z = 0.93948 \]

For not too high pressures where the effect of the 3rd virial coefficient is not predominant, fast convergence can be achieved by iteratively the value of \( v_{it} \):

\[ v_{it} := \frac{R \cdot T}{P} \quad \Rightarrow \quad v_{it} = 3.934 \text{ dm}^3/\text{mol} \]

\[ \frac{P \cdot v}{R \cdot T} = 1 + B \cdot \frac{1}{v} + C \cdot \frac{1}{v_{it}} \]

The solution of interest for \( v \) (and at the same time \( v_{it} \) for the next iteration) of this quadratic equation in \( v \) can be found via "Symbolics, Variable, Solve". The cursor has to be on one of the "v" variable symbols in the equation above:

\[ v_{it} := \frac{1}{2 \cdot P \cdot v_{it}} \left[ R \cdot T \cdot v_{it} + \left( R \cdot T \cdot v_{it} + 4 \cdot P \cdot v_{it} \cdot B + 4 \cdot P \cdot C \right) \right]^{1/2} \]

\[ v_{it} = 3.696157 \text{ dm}^3/\text{mol} \quad \Rightarrow \quad v_{it} - v_m = 3.223 \times 10^{-4} \text{ dm}^3/\text{mol} \]

Solving the equation again with this improved value of \( v_{it} \) leads to nearly the exact result:

\[ v_{it} := \frac{1}{2 \cdot P \cdot v_{it}} \left[ R \cdot T \cdot v_{it} + \left( R \cdot T \cdot v_{it} + 4 \cdot P \cdot v_{it} \cdot B + 4 \cdot P \cdot C \right) \right]^{1/2} \]

\[ v_{it} = 3.695835 \text{ dm}^3/\text{mol} \quad \Rightarrow \quad v_{it} - v_m = 4.644 \times 10^{-7} \text{ dm}^3/\text{mol} \]

In order to find an analytic solution, first apply the operation "Symbolics - Factor"

\[ \frac{P \cdot v}{R \cdot T} - 1 - B \cdot \frac{1}{v} - C \cdot \left( \frac{1}{v} \right)^2 = 0 \]

to yield the following expression:

\[ \frac{\left( P \cdot v^3 - R \cdot T \cdot v^2 - B \cdot R \cdot T \cdot v - C \cdot R \cdot T \right)}{R \cdot T \cdot v^2} = 0 \]